Applied Mathematics and Computation 219 (2013) 7671–7684

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Reactive rimming flow of non-Newtonian fluids

Sergei Fomin [a,*], Anton Mazurenko [b], Ravi Shankar [c], Vladimir Chugunov [d]

[a] *Department of Mathematics and Statistics, CSU Chico, Chico, CA 95929, USA*
[b] *Department of Physics, Harvard University, Cambridge, MA 02138, USA*
[c] *Department of Chemistry, Butte College, Oroville, CA 95969, USA*
[d] *Institute of Mathematics and Mechanics, Kazan Federal University, Kazan, Russia*

ARTICLE INFO

Keywords:
Non-Newtonian
Rimming flow
Numerical analysis
Power-law
Ellis model

ABSTRACT

The steady and non-steady flows of a liquid polymer treated as a non-Newtonian fluid on the inner surface of a horizontal rotating cylinder are investigated. Since the Reynolds number is small and the liquid film is thin, a simple lubrication approximation is applied. Governing equations for non-steady Power-Law and Ellis fluids are solved numerically and the time of transition from non-steady to steady-state mode for various model parameters and flow conditions are defined. The stabilization effect of a chemical reaction within the polymeric fluid (reactive flow) is examined.

1. Introduction

The problem of rotational flow on the inner and/or outer wall of a hollow horizontal cylinder has been of interest for many years due to its wide range of applications in industry [1,2]. Moffatt [3] was the first to derive the condition for the maximal supportable load for a Newtonian liquid. Preziosi and Joseph [4] later presented the same result in another form and named it a run-off condition for coating and rimming flows.

The possible instability of the liquid film on a cylindrical surface is one the most challenging aspects of this problem. The highly unstable nature of rimming Newtonian flow was discussed in a number of publications [5–11]. Although the aforementioned investigations highlight the main characteristics of rimming flow, not enough has been done to show the effect of non-Newtonian properties on such flow, given its importance. Only a few attempts have been made. The power-law [12], Carreau–Yasuda [13], Ellis [14,15], Bingham [16], and viscoelastic models [17,18] have been used to study rimming flow. Fomin et al. [13,14] and Fomin [15] recently extended the estimates, originally made by Moffatt [3] for Newtonian fluids, for a generalized Newtonian fluid. Most polymeric solutes used in rotational coating are non-Newtonian liquids, which exhibit shear-thinning behavior for moderate to high shear rates. Liquid polymers behave as Newtonian liquids near the free surface (at very low shear rates) and exhibit non-Newtonian characteristics above a certain transitional shear rate, $\dot{\gamma}_t$. The importance of non-Newtonian effects for generalized Newtonian fluids is characterized by the shear-thinning number $Wi = \lambda\Omega/\delta$, where Ω is the characteristic angular velocity of the rotating cylinder, δ is the ratio of the characteristic thickness of the film and the radius of the cylinder, and $\lambda \approx (\dot{\gamma}_t)^{-1}$ is a typical time scale for liquid polymers, which is well documented [19] and normally stays in the interval of $(10^{-2}, 10^{-1})$ seconds. In some situations, e.g. a higher speed of rotation, a thinner liquid layer, or a smaller transition shear rate, the value of Wi can be quite large. This illustrates the dominating role of non-Newtonian effects. Results of numerical computations available in [13] show that a Carreau fluid exhibits power-law behavior for large values of Wi. For instance, with $Wi = 8$, the results obtained with the Carreau and power-law models practically

* Corresponding author.
E-mail address: sfomin@csuchico.edu (S. Fomin).

0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2013.01.055